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Water Resources Research

RESEARCH ARTICLE

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Key Points:

- Predicting flow resistance using a bed grain size percentile is unreliable in shallow flows over coarse stream beds
- The alternative of calibrating roughness height in a logarithmic or variable-power resistance equation is tested using data from eight gravel/cobble/boulder reaches
- Roughness calibration at a single intermediate flow depth greatly improves predictions of velocity at higher and lower depths

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Roughness Calibration to Improve Flow Predictions in Coarse-Bed Streams

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Abstract Logarithmic and variable-power equations that use the bed D_{84} grain size as a roughness metric reproduce the general trend of flow resistance in streams with coarse beds, but they are unreliable for predictions in individual reaches. For site-specific application of these equations, I propose that an effective roughness height can be calibrated by making a single flow measurement. I test this idea using published velocity-depth data for eight coarse-bed reaches of varied character. In 52 trials (8 reaches \times 2 equations \times 3 or 4 alternative calibration measurements), single-measurement calibration reduced the root-mean-square error in predicting velocity at all depths by up to 79% (median 66%) compared to using D_{84} . This approach may be useful when prescribing environmental flows, estimating bankfull discharge, or predicting bedload transport in coarse-bed channels in which Manning's n is likely to vary considerably with discharge.

1. Introduction

Predictions of how velocity and discharge vary with flow depth in a stream or river are required for a wide variety of scientific and practical purposes. The extremes of high and low flow are often of particular interest. Flood risk assessment depends on knowing the bankfull discharge, and trash lines or other stage indicators can be used to estimate the magnitude of ungauged floods. Predicting how velocity varies with depth is important when prescribing minimum flows for aquatic habitat protection. The closely related problem of predicting depth (and thus shear stress) from discharge is important in geomorphology because bedload transport is usually predicted from shear stress (e.g., Recking, 2013). In steep streams, bedload predictions may be improved by knowing the total flow resistance as well as the flow depth (Schneider, Rickenmann, Turowski, Bunte, & Kirchner, 2015).

Estimating velocity or discharge from depth, or depth from discharge, requires the use of a flow resistance equation. The most widely used choice is the Manning equation:

$$v = \frac{R^{2/3} S^{1/2}}{n} \quad (1)$$

in which v denotes the cross-sectionally averaged mean water velocity (m s^{-1}), R is the hydraulic radius (m), n is Manning's roughness coefficient ($\text{s m}^{-1/3}$), and S is the energy slope, often approximated by the water-surface slope. The appropriate value of n is sometimes assumed on the basis of textbook tables or channel appearance, or estimated from a bed grain diameter D , but in site-specific applications an alternative that still requires only one site visit is to measure R , S , and v at any convenient discharge and then determine n using Equation 1. This calibrates n to the reach concerned. If n and S can be assumed invariant with stage, and the full channel cross section has been surveyed, the equation then predicts how rapidly velocity and discharge increase at higher flow levels and decrease at lower levels. It can also be inverted to predict depth and velocity from discharge.

Calibration of Manning's n needs to be used with caution in relatively small channels with cobble or boulder beds because n varies with stage in these conditions, sometimes by more than a factor of two (e.g., Ferguson, 2010; Jarrett, 1984). In this study, I consider the possibility that more reliable predictions can be achieved by calibrating the roughness height k in generalized versions of two other widely used resistance equations for coarse-bed rivers: logarithmic and variable-power. The basic concept is illustrated in Figure 1.

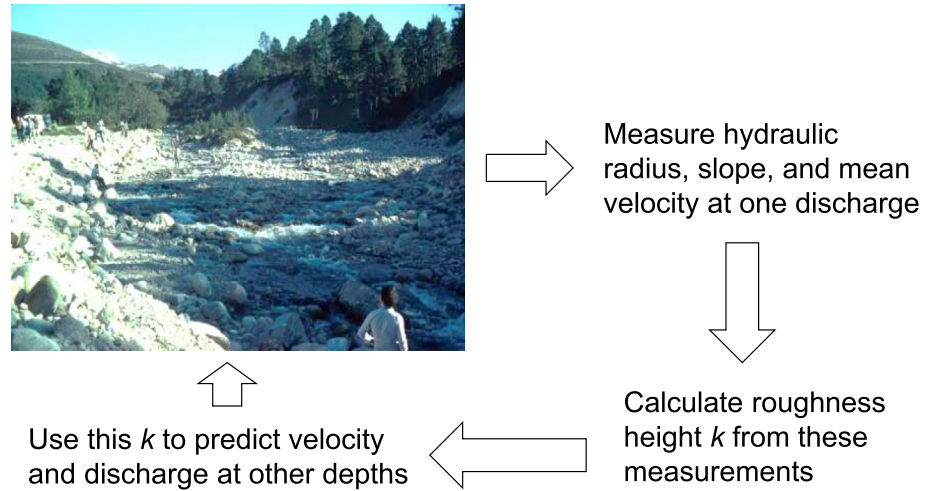


Figure 1. The principle of roughness calibration using a single flow measurement.

To test the potential of this approach, I applied it to flow measurements in eight reaches of contrasting size, gradient, and bed morphology. For each combination of reach and resistance equation, I quantified how well the full array of velocity measurements at different depths was reproduced using a roughness height that was (a) scaled on D_{84} , (b) calibrated using the full set of flow measurements, and (c) calibrated using a single flow measurement at an intermediate depth. Step (c) was repeated for alternative choices of calibration measurement as a test of whether single-measurement calibration gives a consistent improvement over the default scaling of k on D_{84} , and to assess the consistency of the calibrated k values for each reach. At the end of the paper, I discuss how the approach can be adapted for situations where the interest is in partitioning discharge into depth and velocity.

2. Background

The assumption that Manning's n is a constant for a particular reach is equivalent to using the dimensionally balanced flow resistance equation:

$$C_f = \left(\frac{8}{f}\right)^{1/2} = \frac{v}{(gRS)^{1/2}} \propto \left(\frac{R}{k}\right)^{1/6} \quad (2)$$

in which C_f and f are the nondimensional Chézy and Darcy-Weisbach resistance coefficients, g is the gravity acceleration, k is a bed roughness height, R/k is the relative submergence of characteristic roughness elements, and $n \propto k^{1/6}$. This 1/6-power relation is suitable for many rivers, but in small coarse-bed streams it nearly always gives a poorer fit to measurements than is obtained using either of two other relative-submergence equations: logarithmic and variable-power.

The logarithmic resistance relation considered below is the one that Keulegan (1938) derived by integrating the logarithmic velocity profile for turbulent flow over a rough boundary:

$$C_f = \left(\frac{8}{f}\right)^{1/2} = \frac{v}{(gRS)^{1/2}} = \left(\frac{1}{\kappa}\right) \ln\left(c \frac{R}{k}\right) \quad (3)$$

in which $\kappa \approx 0.4$ is von Karman's constant. On an infinitely wide surface the integration constant takes the value $c = 30/\exp(1) = 11.0$, but values up to 13 are appropriate for finite-width channels of progressively lower width-depth ratio (Hey, 1979; Keulegan, 1938); I adopt Hey's (1979) suggestion of $c = 12.2$ for gravel-bed rivers. The consequences of uncertainty in c are minor, as discussed later. The sand-roughness experiments of Nikuradse (1933) suggest that for relatively deep flow over a plane bed of uniform sediment the roughness height k is equal to the grain diameter D , but in coarse-bed rivers the median grain diameter (D_{50})

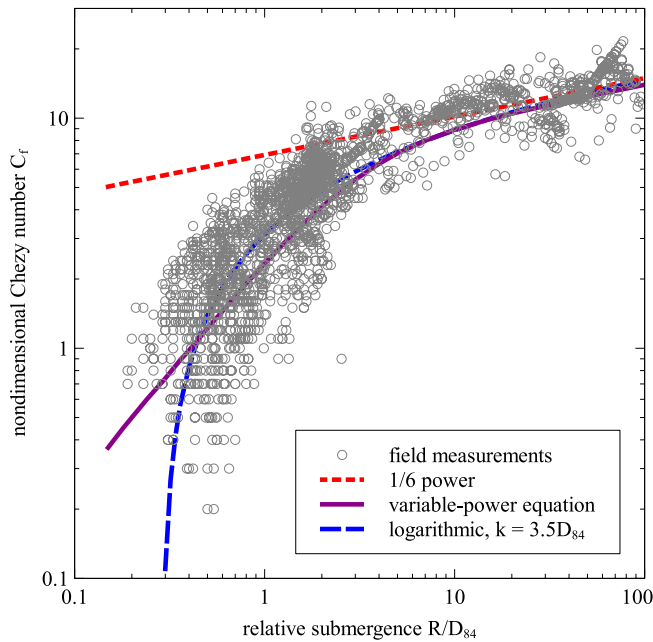


Figure 2. Nondimensional plot of within-reach and between-reach variation in velocity with relative submergence in coarse-bed streams. Author's plot using 2,183 data points from the compilation analyzed by Rickenmann and Recking (2011). Curves are trends predicted by different flow resistance equations.

greatly underestimates resistance. Instead, k is usually scaled on the 84th or 90th percentile diameter (D_{84} , D_{90}) on the assumption that the main energy loss is in the turbulent wakes shed by protruding coarse grains (Clifford et al., 1992). A further complication in shallow flows is that the vertical velocity profile is no longer logarithmic (e.g., Nikora et al., 2001). Despite this, Equation 3 gives a good fit to data compilations from coarse-bed rivers so long as k is set to a multiple of D_{84} or D_{90} . The best empirical fits are with k/D_{84} in the range 3–4 (Bray, 1980; Ferguson, 2007; Hey, 1979; Lopez & Barragan, 2008). I use 3.5 as a default value below.

The variable-power flow resistance equation (VPE) was proposed in Ferguson (2007) as an alternative resistance equation specifically for coarse-bed rivers. Its generalized form is

$$C_f = \left(\frac{8}{f} \right)^{1/2} = \frac{v}{(gRS)^{1/2}} = \frac{a_1 a_2 \left(\frac{R}{k} \right)}{\left[a_1^2 + a_2^2 \left(\frac{R}{k} \right)^{5/3} \right]^{1/2}} \quad (4)$$

with suggested coefficient values $a_1 = 6.5$ and $a_2 = 2.5$ when k is equated with D_{84} . At high values of R/k (deep flows, low relative roughness) the VPE is asymptotic to a Manning-type relation (Equation 2 with prefactor a_1) and gives similar predictions to the logarithmic relation. In very shallow flows, it is asymptotic to the linear relation $C_f = a_2 R/k$ that has been proposed for roughness-layer flow by several authors (e.g., Aberle & Smart, 2003; Nikora et al., 2001; Rickenmann, 1991). Rickenmann and Recking (2011) tested the VPE, Hey's (1979) version of Equation 3

with $k = 3.5D_{84}$, and other alternatives using a very large compilation of flow measurements in coarse-bed streams and found that the VPE was marginally the best at predicting velocity from depth and D_{84} .

Equations 3 (log law) and 4 (VPE) with k based on D_{84} both give more or less unbiased fits to the general trend of large compilations of measurements in coarse-bed streams (Figure 2), unlike Equation 2 (Manning-type 1/6 power) which deviates progressively at low submergence. However, there is proportionately greater scatter about the trend at lower submergence. At $R/D_{84} < 2$ the measured mean velocity can be at least twice, or less than half, the typical value. Some of this scatter is probably due to the difficulty of making precise measurements of depth and grain size in shallow streams with coarse beds. Bed level can be hard to define in boulder-rich reaches. Size-by-number grain size distributions can differ systematically according to the sampling and measurement protocol (Bunte et al., 2009), and percentiles estimated from small samples (commonly the case for boulder beds) have wide confidence intervals (Eaton et al., 2019). For these reasons the R/D_{84} ratio of ~ 1 in a half-meter-deep stream with a boulder bed is far more uncertain than the ratio of ~ 50 in a 2-m-deep river with a pebble bed. More fundamentally, though, scatter around the general trend is inevitable because the small-scale topographic roughness of river beds with the same surface grain size distribution can differ substantially depending on how the grains are arranged: imbricated or not, planar or organized into stone nets or steps. This may in turn depend on sediment supply and recent flow history (e.g., Church et al., 1998). The result is that D_{84} , or any specific multiple of it, is not a reliable proxy for the effective roughness height in any particular reach, despite the overall lack of bias suggested by Figure 2. Calibration of k by means of a single flow measurement is a way to allow for the characteristics of a particular river bed, rather than relying on a general scale relation between k and D_{84} .

3. Test Data

I tested the concept using publicly available data from eight reaches for which flow measurements are available for at least five different in-bank discharges, none of them significantly affected by bank vegetation or large woody debris. The selected reaches span a wide range of channel size, slope, and morphology (Table 1).

Table 1
Flow Measurements Used to Test the Calibration Procedure

Reach name	Median slope	D_{84} (m)	Range of R/D_{84}	Width (m)	Velocity (m s^{-1})	Manning's n ($\text{s m}^{-1/3}$)
Arkansas R	0.023	0.79	1.2–2.1	21–24	1.1–2.6	0.086–0.142
Beaver Kill	0.0043	0.52	0.9–4.9	53–68	0.6–3.8	0.034–0.062
Indian R	0.012	0.55	0.8–1.4	14–19	0.5–1.5	0.064–0.129
Unadilla R	0.0010	0.17	6.3–10.9	45–48	0.9–1.5	0.030–0.039
SLC4	0.016	0.18	1.0–2.3	6–8	0.6–1.6	0.023–0.082
SLC4a	0.016	0.17	1.1–2.3	6–9	0.6–1.6	0.031–0.072
SLC5	0.035	0.54	0.3–0.7	5	0.6–1.8	0.063–0.129
Halfmoon Ck	0.014	0.14	1.4–3.3	9	0.4–1.3	0.055–0.109

The Arkansas River site is from Jarrett (1984, 1985) and is in the Rocky Mountains, Colorado, USA. The Beaver Kill, Indian River, and Unadilla River sites are from Coon (1998) and are in the Catskill Mountains, New York state, USA. The Arkansas River, Beaver Kill, and Indian River sites have more or less planar boulder-strewn beds according to the photographs and cross sections in the original publications. The Unadilla River reach has a lower gradient and a cobble bed. Flow measurements in these four reaches followed USGS protocols and involved surveying and current metering at 3–5 sections per reach for calculation of the reach-average mean depth, hydraulic radius, water-surface slope, energy slope, and Manning's n .

The other four sets of measurements are from sites at which bedload transport has been monitored by or for the US Forest Service in headwater streams in the Colorado Rocky Mountains. Three of them (SLC4, SLC4a, and SLC5) are close together along St Louis Creek and are described in Ryan et al. (2002). They differ in bed character: SLC4 has a plane cobble bed, SLC4a a pool-riffle cobble bed, and SLC5 a step-pool boulder bed. In each of these reaches around 100–200 discharge measurements were made by current meter at a single pool-tail section (Ryan et al., 2002).

The remaining site, Halfmoon Creek, is between sites 2 and 3 of Mueller and Pitlick (2005) and is described briefly in Bunte and Swingle (2021). It has a gravel/cobble bed with pool-riffle morphology and the current-meter flow measurements are from a single pool-tail section. The published data do not include hydraulic radius but the banks are vertical allowing R to be estimated from width and depth. At this site, unlike the others, slope was not measured at the time of every flow measurement. This is a potential source of error in my calculations, since water-surface slope in a pool tail typically increases with discharge.

The D_{84} values at all eight sites are bed-surface values estimated from the measured sizes of a sample of stones. Relative submergence is very low (maximum around 1 or 2) in most reaches but extends to ~ 5 in Beaver Kill and ~ 10 in Unadilla River. Manning's n decreases systematically with discharge in all eight reaches, by about a factor of two over the range of measurements in all but the Unadilla River which has relatively deeper flows than the other reaches. Water-surface slope varies substantially but quasi-randomly in the three St Louis Creek reaches and at the Arkansas River site. It increases slightly with discharge in the Beaver Kill and Indian River reaches, and decreases slightly in the Unadilla River reach.

4. Calibration Procedure

Spreadsheet calculations were done for each reach after arranging the total of N available flow measurements in rank order of discharge. In practical applications, the roughness height k in Equation 3 (log law) or 4 (VPE) could be calibrated directly, but I chose to write the submergence ratio R/k as R/D_{84} divided by a calibration factor k/D_{84} . A baseline fit without calibration was obtained by setting k/D_{84} to 3.5 for the log law and 1 for the VPE. Each measured velocity was predicted from the associated values of hydraulic radius R and water-surface slope S , and the difference between predicted and observed velocity was calculated. The overall goodness of fit of the uncalibrated equation was summarized by the mean and root-mean-square (rms) prediction errors. An overall calibration was then done by optimizing the k/D_{84} factor to give minimum rms error.

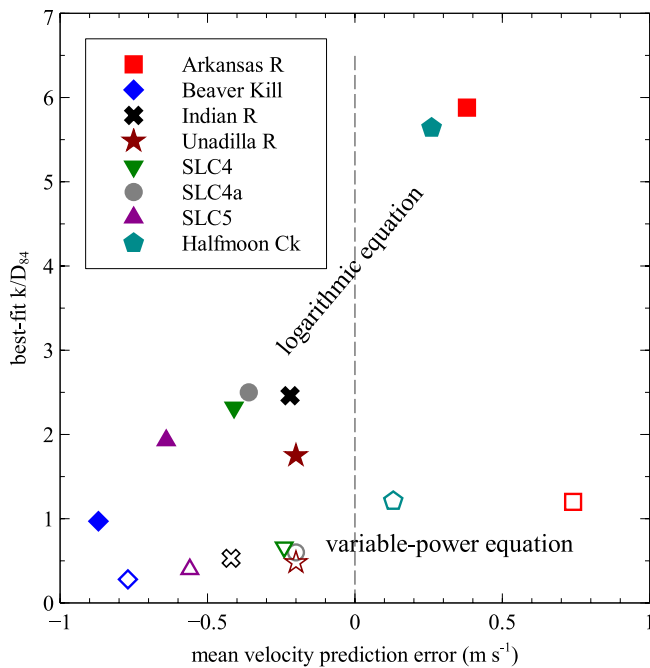


Figure 3. Correlation between mean prediction error of uncalibrated resistance equations and required degree of adjustment to k/D_{84} for minimum root-mean-square error. Solid symbols are for the log law and open symbols are for the variable-power flow resistance equation.

Calibration by means of a single flow measurement was tested using measurements at predefined positions within the array ranked by discharge. In a practical application concerned with flood flows, it would be desirable to calibrate using as high a measured flow as possible, and the fit to low flows would be irrelevant. Conversely, in an application concerned with low flows it would be desirable to calibrate using a low flow. Rather than give detailed worked examples of these two cases separately I present a general proof of concept here, using calibration flows distributed over most of the range of discharge and assessing the goodness of fit to the entire range of flows.

In the three SLC reaches and Halfmoon Creek, for which a large number of measurements are available ($N = 30\text{--}207$), calibration was done using the measurements one quarter, one half, and three quarters of the way down the discharge-ranked list. Far fewer measurements are available for the other four reaches. Calibration was done using the middle three of the $N = 5$ within-bank measurements available for the Arkansas River reach, those ranked 3, 5, and 7 of $N = 9$ in Beaver Kill, and those ranked 3, 5, 7, and 9 of $N = 11$ in the Unadilla River and Indian River reaches. For each chosen calibration measurement, k/D_{84} was optimized to predict correctly the measured velocity using the measured hydraulic radius and water-surface slope at that discharge. The same slope was then used with the calibrated k/D_{84} ratio to predict the other $N - 1$ measured velocities, thus simulating extrapolation from a single field measurement to the full range of higher and lower discharges. Predictive accuracy was again quantified by the mean and rms prediction error calculated using all N predictions.

5. Results

The uncalibrated log law and VPE give biased predictions of velocity in all eight reaches. Both equations systematically overpredict measured velocities in the Arkansas River and Halfmoon Creek and underpredict them in the other six reaches (Figure 3). This is equivalent to underestimating the effective bed roughness of the Arkansas River and Halfmoon Creek but overestimating it at the other sites. Mean prediction errors are greatest in the two reaches (Beaver Kill and Arkansas River) in which velocities are highest. When expressed in relative terms, mean prediction errors are mostly in the range of 10%–30% of the maximum measured velocity in the reach concerned.

Calibrating either equation by minimizing the rms error of predictions of all N measurements in a reach almost completely eliminates the overall bias of the uncalibrated equations: mean prediction errors are reduced in every case to less than $\pm 0.03 \text{ m s}^{-1}$, which is typically less than 1% of the maximum measured velocity. As might be expected, the required degree of adjustment to k/D_{84} increases with the mean prediction error of the uncalibrated equation (Figure 3). As can be seen from Equation 3, the best fit values of k would be a few per cent higher or lower if a different value was assumed for the integration constant c , but the predicted velocities are unaffected by this. Calibrated values of k/D_{84} for the log law range from 1.0 to 5.9, and those for the VPE from 0.3 to 1.2, compared to the default values of 3.5 and 1, respectively. There is no obvious link between reach morphology and the direction and degree of bias. Single-point calibration also reduces the prediction bias for all 52 combinations of reach, prediction equation, and choice of calibration measurement. The improvement is generally considerable, and sometimes as much as with full calibration.

A small mean prediction error does not necessarily indicate a good fit to data: it could be achieved by systematic overprediction of velocity at low discharges and underprediction at high discharges, as might well happen if Manning's n was calibrated at an intermediate depth and assumed constant. A good overall fit also requires a small rms error. Figure 4 shows the full results using this metric. The uncalibrated fits with high overall bias inevitably also have high rms errors (approaching 1 m s^{-1} in one reach) and the overall median

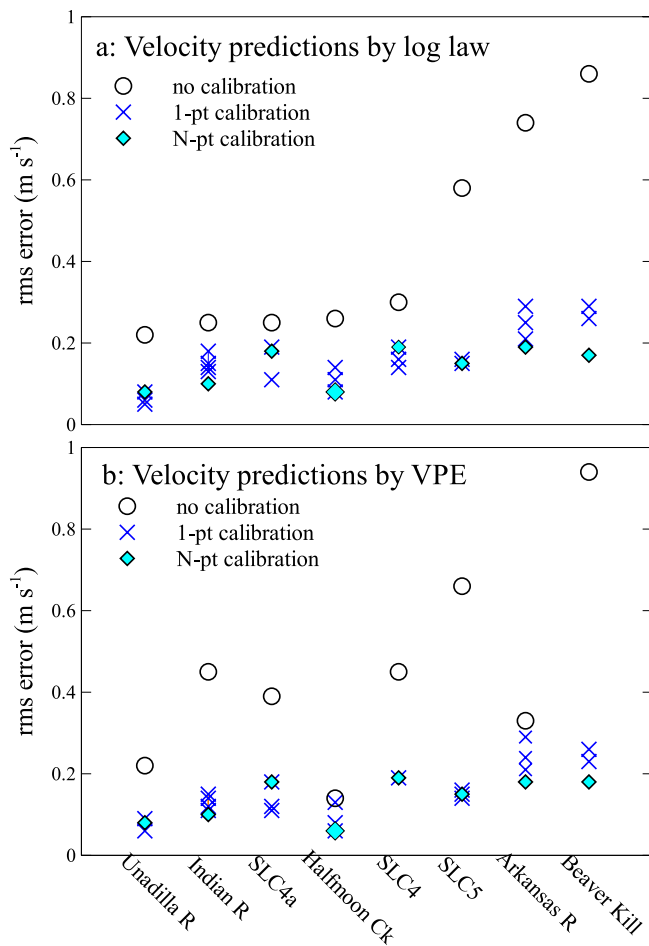


Figure 4. Root-mean-square error in predicting velocity from hydraulic radius using (a) log law or (b) variable-power flow resistance equation (VPE), showing improvement after calibration of roughness height using a single measurement (1-pt) or all measurements (N -pt).

rms error is quite high at $0.36 m s^{-1}$. The fully calibrated fits are far better, with rms errors of 0.08 – $0.19 m s^{-1}$ for the log law and 0.06 – $0.19 m s^{-1}$ for the VPE (overall median $0.15 m s^{-1}$). This confirms that either of these flow resistance equations is appropriate for use in these eight channels. The full-calibration rms errors also give an indication of the maximum possible improvement in predictive ability after single-point calibration. It would be a precise limit if water-surface slope was invariant with discharge in each reach: the single-point estimate of k would either coincide with the overall best fit k , giving the same rms error, or be different and give a higher rms error.

Calibrating roughness height using a single measurement gives a big improvement in rms error over the uncalibrated equation in 12 of the 16 combinations of reach and resistance law (Figure 4). The exceptions are where an uncalibrated equation already fits fairly well: the logarithmic law in Indian River and SLC4a, and the VPE in Arkansas River and Halfmoon Creek. The median reduction in rms error across all eight reaches is 62% with the log law and 69% with the VPE (overall median 66%). Some of the single-point calibrations for Unadilla River and the three St Louis Creek reaches give an even lower rms error than after calibration using the full N -point data set. This unexpected finding is related to the substantial variation of water-surface slope at these sites. The prediction errors are strongly correlated with the slope measurements, with a tendency for overprediction at higher slopes and underprediction at lower slopes via the term $(gRS)^{1/2}$ in the prediction equation. This suggests that some of the slope values differ from the true energy slope or are subject to measurement error. Overall, single-point calibration gives comparable predictive accuracy to calibration using the entire data set: with either equation the mean and median rms errors across all eight reaches are 0.16 and $0.15 m s^{-1}$, respectively, compared to 0.15 and $0.18 m s^{-1}$ for full calibration.

The choice of resistance equation makes little difference to the improvement in velocity prediction that is gained by single-point calibration. In eight cases, the rms prediction error after calibrating the log law is the same (to the nearest $0.01 m s^{-1}$) as after calibrating the VPE. The log law rms error is marginally lower in six cases and the VPE rms error marginally lower in 11.

An example of the improvement single-point calibration can offer is given in Figure 5, using results for reach SLC5. The log law and VPE give similar results in this reach, so only the former is shown, but results using calibration of Manning's n are included for comparison. The 93 measurements show a well-defined trend of increasing mean velocity with hydraulic radius, but with substantial scatter. The single measurements used for calibration (upper quartile, median, and lower quartile in the list ranked by discharge) are highlighted in Figures 5a and 5c. The uncalibrated VPE grossly underpredicts the measured velocity at these discharges, but calibration of roughness height to match any one of these three points gives a good overall fit to the trend of the data (Figures 5a and 5b). In contrast, calibrating Manning's n using the same three measurements gives different fits depending which measurement is used, and the fits are systematically skewed (Figures 5c and 5d): velocity is overpredicted at low discharges but underpredicted at high discharges.

Roughness calibration using the entire data set yields a single best fit value of the k/D_{84} ratio for a particular reach, but single-point calibration yields a different best fit value for each choice of calibration measurement. These values are plotted in Figure 6 in order to examine their consistency. There are two questions here: how consistent are the different estimates of k/D_{84} for a particular reach and equation? And how consistent are the adjustments required to k/D_{84} according to which equation is used to predict velocity?

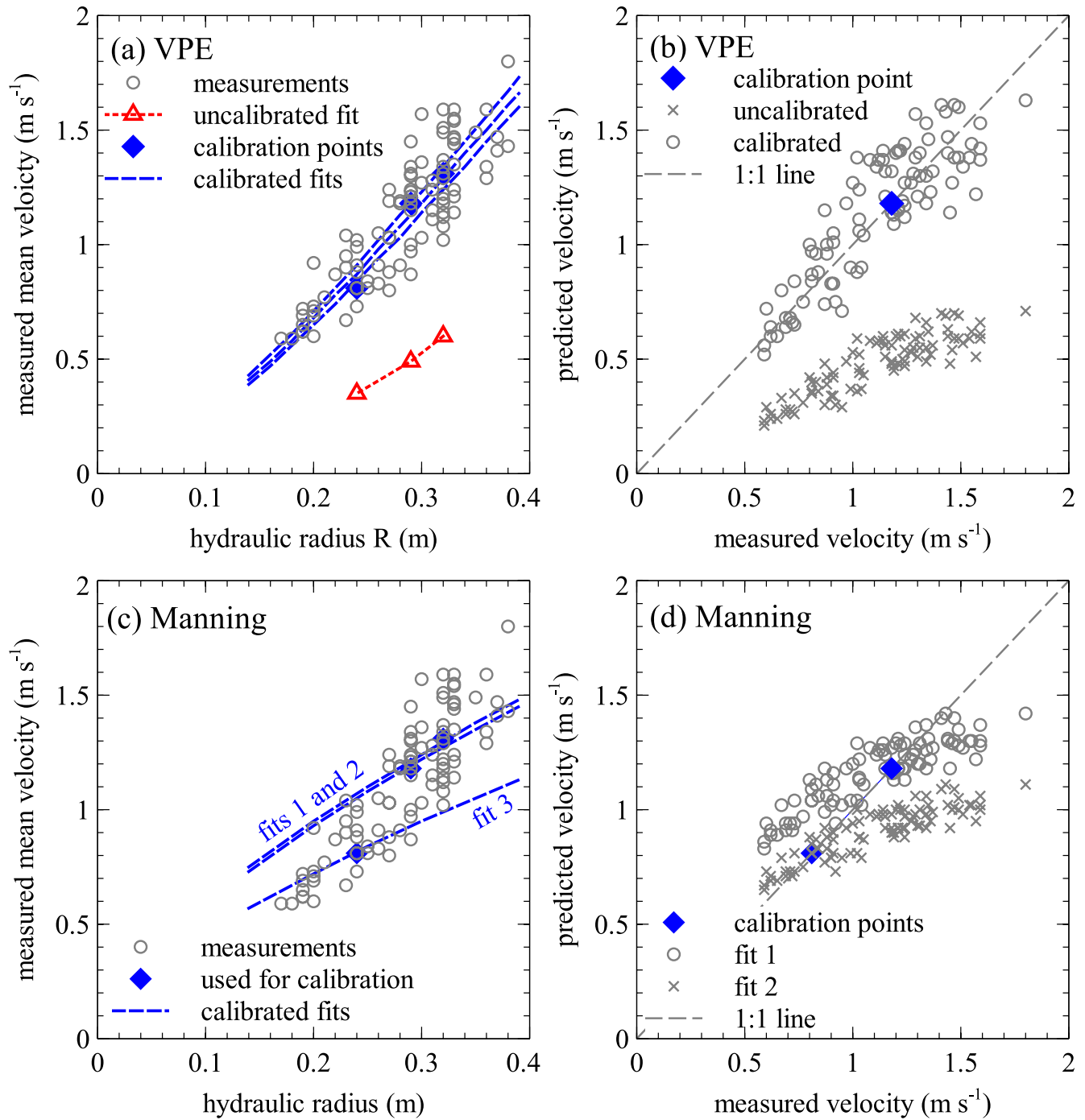


Figure 5. Single-point calibration of (a and b) VPE and (c and d) Manning equation in reach SLC5. Parts (a and c) compare 93 paired measurements of mean velocity and hydraulic radius with fits calibrated using each of three highlighted measurements; (a) also shows the uncalibrated variable-power flow resistance equation (VPE) predictions for those three measurements. Parts (b) and (d) show agreement between predicted and measured velocity.

The first relates to the sensitivity of the results to the choice of calibration measurement, the second to sensitivity to the choice of equation. It is also interesting to see how far the best fit k values deviate from the defaults of D_{84} (VPE) or $3.5D_{84}$ (log law).

The scatter of best fit values of k/D_{84} according to the choice of calibration measurement is much less than the systematic difference between reaches (Figure 6). The k/D_{84} values for five of the reaches are tightly grouped, and those for Arkansas River and Unadilla River are only slightly more dispersed. The wider

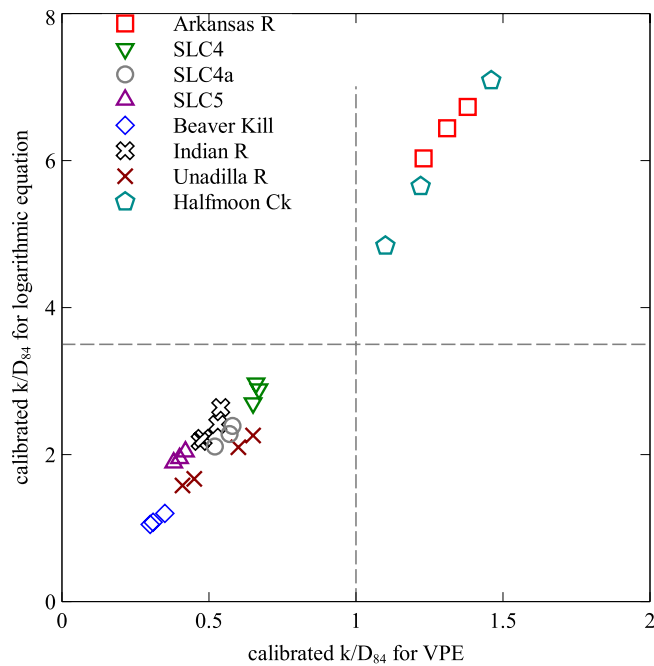


Figure 6. Results of single-point calibration of k/D_{84} in the log law and variable-power flow resistance equation (VPE), showing departure from default values but consistency within reaches and between equations. Dashed lines show the default values of k/D_{84} for each equation.

spread for Halfmoon Creek may be related to the lack of data on how water-surface slope varies with discharge; if it increases, as might be expected at a pool-tail measurement transect, the estimates of k/D_{84} would be less dispersed. As regards consistency between equations, a strong positive correlation is apparent between the best fit values for the log law and VPE, both overall and within the results for individual reaches. In general, the calibrated k/D_{84} ratio for the log law is about four times higher than for the VPE in the same reach. As with the full-calibration results in Figure 3, the reaches differ greatly in how well D_{84} represents roughness: the best fit k/D_{84} ratios range from 0.3 to 1.6 for the VPE, and from 1.0 to 7.1 for the log law. The latter range is far greater than the previously mentioned uncertainty of a few per cent in log-law k values according to the choice of the integration constant in Equation 3.

6. Discussion

Treating Manning's n as a constant property of a reach must be done with caution in small rivers with coarse beds, where n usually declines considerably with increasing discharge. This is not a problem if predictions are only required for flow levels close to the one used for calibration, but extrapolation to much higher or much lower levels generally leads to systematically skewed predictions at other discharges, as demonstrated in Figure 5d. The results presented here show that single-measurement calibration of a logarithmic or variable-power resistance equation may be a more reliable alternative. Both of these equations, when calibrated, give good overall fits to velocity measurements in the eight reaches studied here. The approach may be useful for a variety of applications that

require prediction of velocity or discharge, whether at high stages (e.g., bankfull) or low (e.g., minimum prescribed flows). If grain size data are available, calibration can be done by adjusting the k/D_{84} ratio away from its default value (as in my tests), but it is equally possible to calibrate k directly in Equation 3 or 4 starting from a trial value.

The approach taken here using calibration of roughness height is not the only possible way to modify the original versions of these equations. Some previous authors (e.g., Bathurst, 1985) used wide-ranging data sets to calibrate a generalized log law of the form $C_f = c_1 \ln(c_2 R/D_{84})$ with c_1 and c_2 treated as fitting parameters, whereas I have kept $c_1 = 1/\kappa$ and assumed $c_2 = 12.2$. Similarly, Schneider, Rickenmann, Turowski, and Kirchner (2015) adjusted the empirical coefficients a_1 , a_2 in the VPE to obtain a better overall fit to a data compilation. The present approach has two advantages: it has a clearer conceptual basis (D_{84} is an unreliable proxy for k) and is simpler to apply to a specific reach (calibrate one parameter rather than two at once).

Other equations altogether could be calibrated to individual reaches. One such is the equation proposed by Jarrett (1984) for Manning's n as a function of R and S . This does not contain a roughness coefficient, but can be calibrated to a site by using a single gauging to adjust the prefactor in his regression equation. I tried this in two of the reaches considered here, including Arkansas River which was one of the reaches to which Jarrett fitted his equation, but the rms errors were systematically higher than those obtained by single-measurement calibration of the logarithmic and VPE relations.

An intriguing feature of the results in Figures 3 and 6 is that calibration reduces k in six of the eight reaches, rather than giving a more symmetric balance between increases and decreases in k . This finding may just be chance, since the default values of k give good overall fits to large data compilations (Figure 2). It does not appear to be related to hydrological regime or sediment supply: the five Rocky Mountain sites are dispersed across Figure 6 despite their geological and hydrological similarity. It would nevertheless be interesting to calibrate k to measurements from a wider range of sites to explore this further. Whatever the explanation for this asymmetric outcome, the wide range of calibrated k/D_{84} values does support the argument made throughout this paper that D_{84} , or any other bed grain size, is an unreliable proxy for the effective roughness

height of a stream bed. The small-scale topographic irregularity of the bed is what retards the flow, and it depends on grain organization as well as grain size distribution. Aberle and Smart (2003) found that the standard deviation (s_z) of bed elevation variation along high-resolution longitudinal profiles of a planar flume bed gave better fits than D_{84} in the generalized log law. In a recent study, Chen et al. (2020) found s_z outperformed D_{84} with each of four resistance equations applied to a large data compilation. It would be interesting to investigate the extent to which calibrated values of k/s_z vary between natural reaches with coarse beds and no large woody debris.

The tests reported here are for prediction of velocity, and thus also discharge, from depth. This is what is required in some applications, but in others the requirement is to partition a known or assumed discharge into its components of depth and velocity or to predict shear stress from discharge. There are several ways to use single-point calibration for the latter type of application. If all that is needed is a table or graph of how depth and velocity vary with discharge, the approach described above still works: calibrate k to one flow measurement, predict velocity (and thus also discharge) at an array of other depths, and plot R and v against Q . What cannot be done so easily is to calculate R and v from a particular discharge value in the way that can be done using the Manning equation with fixed n . Logarithmic laws, and the original form of the VPE, are not algebraically invertible in this way, so an iterative solution would be required. This is easily done for a site-specific calculation but is inconvenient as part of a general model. A direct calculation is, however, still possible in either of two ways. In very shallow flows it can be done by inverting the shallow-flow asymptote of the VPE ($C_f = a_2 R/k$). More generally, the VPE-based nondimensional hydraulic geometry equation proposed by Rickenmann and Recking (2011) can be manipulated to express depth as an explicit function of unit discharge, slope, and roughness height.

7. Conclusions

This paper has tested how well the variation of velocity with depth in shallow coarse-bedded streams can be predicted using only information from one site visit. The proposed approach is to estimate flow resistance using a logarithmic or variable-power function of relative submergence, R/k , and to use a single flow measurement to calibrate the roughness height k . Normally k is equated with the bed D_{84} for use in the variable-power equation, or with a multiple of D_{84} for use in the logarithmic equation. These scalings on grain size reproduce the overall trend of flow resistance in large data compilations, but are unreliable at individual sites which may have more or less resistance to flow than is suggested by D_{84} .

The tests use published data on the variation of velocity with depth and discharge in eight reaches that differ in size, slope, grain size, and bed morphology. Without any calibration, the logarithmic equation of Hey (1979) with $k = 3.5D_{84}$ and the variable-power equation (VPE) of Ferguson (2007) with $k = D_{84}$ both give moderately to highly biased predictions of velocity. After calibration of the ratio k/D_{84} using the full set of available measurements in a reach, both equations give good fits to every reach, confirming their suitability for shallow flows. In contrast, Manning's n is far from invariant with discharge in seven of the eight reaches.

Calibrating k/D_{84} to match a single measured velocity at an intermediate discharge gives more accurate predictions of how velocity varies with depth in every reach. The overall median rms error in predicted velocity is 0.15 m s^{-1} , compared to 0.36 m s^{-1} when using D_{84} . In four of the 16 combinations of reach and equation, the improvement is modest because one of the uncalibrated equations was already a fairly good fit. In the other 12 cases, single-point calibration gives a reduction of up to 79% in the rms prediction error. In many cases, single-point calibration is as effective as, or even more effective than, calibration using the full data set. There is no clear indication in the results that one equation should be preferred over the other.

Three or four single-point calibrations of each equation were made in each reach as a check on consistency. The best fit k/D_{84} ratios are fairly consistent both within each reach and between equations. They range from 1.0 to 7.1 for the log law and from 0.3 to 1.6 for the VPE. This supports the premise of the paper, that D_{84} (or any other bed grain size) is an unreliable proxy for the effective roughness height of cobble/boulder channels.

In practical applications, only a single calibration measurement is required. It should preferably be at a fairly high discharge if the interest is in flood flows, or a fairly low discharge if the interest is in low flows, but the results in Figures 4–6 suggest that calibration anywhere in the discharge range is likely to give an improvement over the uncalibrated equations. The tasks to be performed during the single site visit are (a) survey the channel cross section, (b) obtain the best possible estimate of the energy slope S , and (c) measure the mean velocity by the most convenient method: typically current metering, but possibly dilution gaging or surface velocity measurement by a near-field remote sensing method. The roughness height k is then estimated from the known slope, hydraulic radius, and mean velocity. These values of S and k are then used to predict velocity and discharge at other flow depths.

This calibration tactic may be useful for a variety of applications that involve extrapolation to flows that are much higher or much lower than at the time of the single site visit, and it can be adapted to predict depth from discharge. Future work could usefully test that adaptation, and test the concept in additional combinations of reach characteristics.

Conflict of Interest

The author declares no conflicts of interest relevant to this study.

Data Availability Statement

All data used in the paper are publicly available. The flow measurements in the Arkansas River are tabulated in Jarrett (1984, 1985), available at <https://pubs.er.usgs.gov/publication/wri854004>. Those in Beaver Kill, Indian River, and Unadilla River are tabulated in Coon (1998), available at <https://pubs.er.usgs.gov/publication/wsp2441>. The measurements in reaches 4, 4a, and 5 of St Louis Creek are available at <https://en.bedloadweb.com/> which is curated by Alain Recking. The measurements in Halfmoon Creek are summarized in Bunte and Swingle (2021) and are available in full via the “Halfmoon 2015” tab at <https://www.fs.fed.us/biology/nsaec/projects-bedloadtraps-database.html>.

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